Inquiry-Based Argumentation in Primary Mathematics: Reflecting on Evidence

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Argumentation in mathematics teaching has potential to move students beyond tacit understanding of mathematical concepts and procedures towards articulation and justification of their ideas; a practice in which evidence is central. Design-based research was used to examine the nature of evidence used by a class of primary students through levels of argument and explanation. Results of this exploratory study indicate that evidence put forward became increasingly sophisticated as students' conceptions became public and therefore open to increased potential challenge.

The potential to enculturate students into subject-specific practice and discourse is one of the primary benefits to the introduction of explanation and argumentation into classroom environments (Simon & Richardson, 2009). While argumentation can engage students in learning practices that more closely approximate the mature discipline of mathematics, other benefits have been identified and documented within and across subject domains. For example, research into involvement in argumentation practices suggests students develop increasingly secure conceptual understandings (Asterhan & Schwarz, 2007), an increase in context knowledge (Zohar & Nemet, 2002), and the opportunity to develop high levels of subject specific literacy (Jimenez-Aleixandre & Erduran, 2007). One explanation for these findings is that argumentation practices support the 'visibilising' of cognitive processes (Berland & Reiser, 2009), which in turn enables teachers to identify and enhance or challenge student understandings (Jiménez-Aleixandre & Erduran, 2007). Argument in the sense described here does not imply the need to hold forth a "winning" position; rather it involves collaborative discussion to explore and resolve issues in a manner which best fits available evidence and logic (Berland & Reiser, 2009). As such, evidence necessarily takes a central role in argumentation structure and practice. This paper reports on exploratory research which engages young students in inquiry-based argument using a theoretical framework based on the goals of argument (Berland & Reiser, 2009). A central focus is on students' evolving use of evidence.

Literature

While mathematical inquiry has been a topic of much research focus, there has yet to be a consistently agreed upon definition among researchers. For the purpose of the research described here, inquiry is defined as being the addressing of an ill-structured question in which the inherent ambiguity affords opportunities for multiple solution pathways *and multiple solutions* (Makar, 2010; Reitman, 1965). This requires a student to focus on decision making, analysis and justification. Rather than a 'correct' answer or strategy, there is a claim (often qualified) which requires evidence, explanation and defense—in short, an argument.

One of the mostly widely used frameworks for examining argument structure is that of Toulmin, Rieke and Janik (1984). Their argument framework describes multiple elements of an argument. At its simplest level, an argument consists of a claim (the assertion that identifies the stance and position taken), grounds (the underlying support, or evidence, that

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is required to enable the claim to be accepted) and warrant (the justification for moving from grounds to claim). Warrants enable the checking of the grounds, to determine whether they offer genuine support for the claim or whether the grounds are irrelevant or unwarranted.

Considerable prior research into argumentation in mathematics education exists. However, this research has focused largely on mathematical proof (see, for example, Conner, 2007; Lampert, 1990) or argumentation as it applies to procedure (see, for example, Brown, 2007; Dixon, Egendoerfer, & Clements, 2009; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Goos, 2004; Yackel & Cobb, 1996). Lampert asserts that it is *"the strategies used for figuring out, rather than the answers, that are the site of the mathematical argument"* (1990, p. 40). The research described in this paper differs from the existing body of literature somewhat in that both the solution process and the answers are considered the *site of the argument.* Hence, the term *inquiry-based argument* has been adopted.

Theoretical Framework

Berland and Reiser (2009) propose three hierarchical levels of argumentative discourse that are determined by the goals of sense-making, articulation and persuasion. The goal of sense-making is for students to develop a personal understanding of the phenomena under investigation. Sense-making consists of an inwardly focused belief system and therefore may lack a reliable, valid evidence base or stem from an incomplete and unchallenged position. By contrast, when the goal is to articulate to an audience their reasoning and make evident their claim, they are situated in such a way that they need to closely consider their position and supporting reasons, and their need to make their links clear, thus engaging more deeply and critically with the evidence they put forward. The third level of argumentation is persuasion; the goal here is to develop the most robust explanation of the studied phenomena. As the community of learners puts forth their views and evidence, students are required not only to articulate their findings and claims, but also to be able to defend, justify and reflect as they are challenged. This necessitates a deeper understanding of the phenomena and the evidence offered in its support.

Drawing on the framework offered by Berland and Reiser (2009) it was anticipated that, as students progressed through the goals of argument, their cognitive processes would become increasingly public, increasing the potential for challenge, and thus bring about an increased reliance on evidence and quality of evidence. Sampson and Clark (2006) have proposed criteria specifically for examining the quality of students' arguments: the nature and quality of the claim, how far the claim is justified, if the claim accounts for all the available evidence, how the argument attempts to discount alternatives, and how epistemological references are used to coordinate claims and evidence.

Context, Design and Methodology

The findings reported here derive from the early stages of a doctoral research project designed to further understand the teaching and learning of argumentation practices in primary school mathematics. The purpose of the research is to promote deep understandings of, build theories surrounding, and provide practically relevant approaches to the development of young children's argumentative and explanatory practices in mathematics. Specifically, the focus here is on students' early experiences with inquiry-based argument, looking in particular at the role of evidence in the argument. Within this context, the research question addressed is: When engaged in Inquiry-Based Argument, how does the student's focus on evidence evolve as their cognitions become increasingly public?

The research project uses Design Experiment methodology (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) as the aim is to develop theory through multiple iterations of reflective-prospective cycles of improvement. Design Experiment is underpinned by two premises, the need to make the research relevant to practice, and the premise that the classroom is a complex, evolving, dynamic system that cannot be objectively observed. The researcher is not simply an observer, but rather creates interventions to the system which affect the nature of this intertwining and therefore creates a subjective system (Lesh, 2002). In this instance, the researcher has taken on the dual role of research-practitioner in teaching the unit described.

The section of the study reported here is drawn from a single class of Year 4 students (age 8-9) in a suburban state school in Queensland. This class was in its fourth term of the school year and had previously been involved in inquiry learning with both the author and her teaching partner over the previous eight months.

The initial question, "*Is Barbie a human*?" was determined according to school curriculum planning for the year level and was tied to a culminating art project. When the mathematics unit commenced, students had just engaged in a two-week Visual Arts unit exploring proportion of the human face in art.

Data collection occurred through classroom video, collection of student artefacts, researcher reflective journals and research notes. Video recordings were transcribed in full and episodes which illustrated students developing use of evidence were selected for analysis. The results presented in the next section illustrate examples of developing understandings, difficulties and identified supports over the course of the unit.

Qualitative content analysis (Flick, 2009) enabled the video transcripts to be reduced to interactions relevant to this research focus and then coded thematically. Theoretical frameworks used to derive categories included those of Berland and Reiser's (2009) goals of explanation and argument and Toulmin et al's. (1994) argument structure. Both classroom dialogue and student work samples were mapped against these frameworks.

Results

The excerpts presented below are selected for their potential to provide insight into students' developing awareness of the need for defensible mathematical evidence. It is necessary to note that the students use the term 'claim' to identify their position and 'evidence' to refer to both grounds and warrants.

Developing the need for a mathematically researchable question

The initial question "Is Barbie a human?" was provided to the students. In the excerpt below, the teacher's goal is to have the students refine the question to something which has the potential to be mathematically researchable, and which would necessitate conceptual coverage of proportion.

1. Teacher: Now what do we need to think about to know 'Is Barbie a human?'
2. Delmar: No way
3. Teacher: Why not?
4. Delmar: She is made of plastic.

…

5.	Dominica:	She could be kind of looking like her eyes are not halfway from her crown to her chin.
6.	Teacher:	Are they?
7.	Students:	[choruses of yes and no]
8.	Teacher:	Has anyone checked?
9.	Oliver:	We could get a few groups and test them.
10.	Delmar:	And when you look at her she doesn't talk.
	…	
11.	Teacher:	Do we need to narrow down this question a bit?
12.	Dominica:	I think we can all see that she doesn't walk but we are talking about the way she looks.
13.	Teacher:	OK. Do we refine this a bit [pointing to the question]?
14.	Shana:	You mean if she was a human would she be like a human?
15.	Teacher:	In what way?
16.	Konrad:	Like her face is the same as that [pointing to facial proportions diagram from the morning art lesson].
17.	Teacher:	So if she was real, would her face meet these proportions?
18.	Oliver:	If the Barbie girl was real life size would she look [tails off]
19.	Connor:	Does Barbie have the same proportions as a human?

Initially an ungrounded claim is provided [2]. However, students quickly move to providing grounds [4, 5, 10]. This is likely a factor of the students being well-versed in the expectation to explain any answers; a long standing classroom norm across subject disciplines. However, the initial responses were not typically mathematically researchable in this context [4, 10]. As the teacher and students engage in further discussion, the teacher continuously draws students toward the goal of a mathematically researchable question [11, 13]. This exchange was brief, likely because the students had proportion in the forefront of their minds and this enabled the students to make connections to the artwork and the visual conceptions they had of proportion already [14, 16, 18, 19].

After class agreement to adopt Connor's question [19], an immediate shift in responses was noted which clearly reflected the new question [20, 22]. These responses were of an observational nature and the teacher used them to instil the need for evidence [21, 23, 27].

20.	Shana:	Her eyes are too big.
21.	Teacher:	Are they? How do you know that?
22.	Shana:	Well, her neck is not normal, it is too long.
23.	Teacher:	Why do you say that?
24.	Shana:	It looks too long.
25.	Dominica:	No. It looks about normal.
26.	Oliver:	We could test it.
27.	Teacher:	How?
28.	Oliver:	Someone could bring in Barbie dolls and we could get into our groups and we could look at it and we could estimate if she was a human height whether she would be normal.

The teacher attempted to prompt the students to recognise a need for evidence. However, it was when Shana and Dominica disagreed, that a resolution became more important to the class and served to focus students more effectively than the teacher's comments [23, 25, 26]. This minor and amicable disagreement was to resurface repeatedly throughout the unit.

Envisaging the Evidence

With the need for evidence and a mathematical focus identified, students were tasked with developing a plan that would enable them to gather the evidence they would need to be able to answer the question. Without fail, each group of students became mired in the detail of relative irrelevancies such as: which version of Barbie to use, whether Ken and Barbie would have similar proportions, and whether Barbie's measurements would be different because she was plastic. In order to move the students forward, the teacher explicitly discussed the need for claim-evidence links. She then had the students individually draft an 'imaginary' conclusion considering what the claim might be and what evidence they might use to support it. Each student provided a claim modelled on the teacher's example; "Barbie does/does not have human proportion because…". Many students provided more than one reason and when they did so, each reason was considered as a separate statement of evidence. Approximately one third of the statements indicated an explicit focus on proportion (including a hypothesised ratio), a further third indicated that they were envisaging proportion in their evidence focus (implied proportional reasoning) and the remaining third were not focussed on proportion.

Table 1
Examples of Evidence Students Envisaged would Address their Question

Envisaged Evidence	Number	%	Example (from student responses)
Explicit Proportional Reasoning	21	33.9	It's knee isn't halfway down the leg (Connor)
			Halfway down from the nose to the chin is the middle of your mouth and barbie is the same (Cho)
Implicit Proportional Reasoning	18	30.5	Her arms are the same size of a human if we made her to size (Sadie)
Additive	5	8.5	Her feet are 3cm longer than the human proportion (Seth)
Methodological	5	10.2	We measured Barbie and then we measured a human then shrinked the human and put both proportions on a piece of paper and compared (Andrea)
Other	10	16.9	Her ear only has one bump (Konrad)

The students came back together and articulated their envisaged evidence to their groups. As the students talked within their groups, they came to the understanding that the evidence required would be in terms of a proportion, while the students who had envisaged methodology were able to contribute ideas for discussion. This resulted in plans being quickly formulated that had the potential to gather useful evidence:

> Gemma: Our plan. 1. Measure human proportions. 2. Measure Barbie's proportions. 3. Compare Barbie and human proportions and see if they're close. Like measure head size and work out if the eyes are halfway and do same with Barbie and see if the proportions are close.

As each group shared their plans and their complications, it became apparent that they could see what was required while experiencing two principal sources of difficulty. The first was in terms of the practical aspects of comparing human-Barbie proportions, and the second was the uncertainty of many proportions to compare. The students requested instruction from the teacher as to how to proceed mathematically and at this point direct teaching took place using models of unifix cubes both with and without comparison to human features, fractional representation, and use of the algorithm with interpretation of the decimal answers. For practical reasons, the teacher selected the proportions to be considered. Students proceeded to collect a range of measurements from their parents and calculated the proportions (rounded to one decimal place). These results were amalgamated into a class data set and each student was assigned responsibility for working with a single proportion, with their results to be compiled and considered as a whole afterwards. The students' plans indicated a need for first determining what a *normal* human being would look like. The students were asked to complete five successive tasks (each given at the completion of the prior task). These tasks were selected to establish a focus commensurate with the Berland and Reiser (2009) goals of *understanding*, *explanation* and *persuasion*. The tasks were to:

a) determine their 'answer' in their inquiry journals so that they could *understand* it,
b) rate how well they thought their answer would be believed by others,
c) create a poster that would stand alone to *explain* their answer to others,
d) constructively critique each other's posters in terms of how convinced they were by the information, and
e) create a new poster which they thought would *persuade* or *convince* others.

The students could clearly differentiate between each of the requirements. While very few students changed their method of representing their evidence from the *understanding* task to the *explaining* task (Figure 1), a notable difference was that all but two students then included some form of written explanation; typically explaining *how* results were obtained but leaving the reader to interpret the representation. A greater change came after the students had provided feedback to each other in terms of whether individual posters were persuasive and the students' role was to now to *convince.* There was an overwhelming shift towards the use of ordered dot plots (Figure 1) accompanied by claim-evidence-reasoning statements. Many of the students also took the opportunity to incorporate additional information into their poster designed to convince. For example, students attempted to explain outlying data using, as a reference, what could be realistically expected in terms of human proportion.

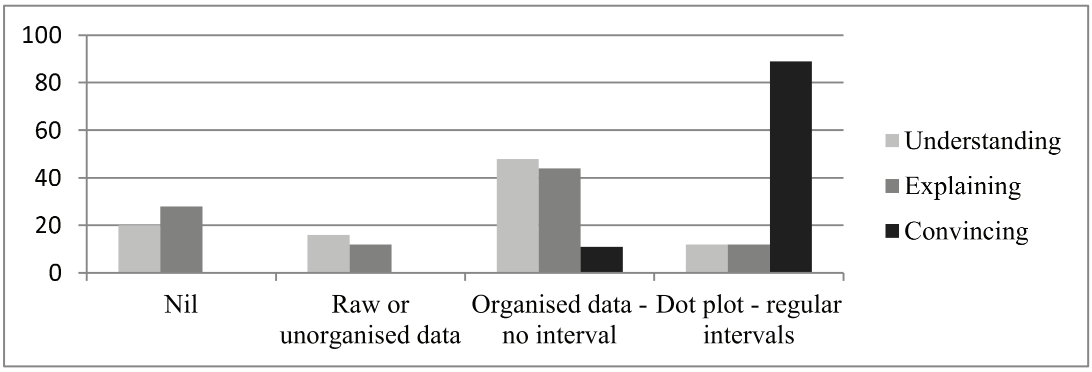

Figure 1. Forms of data representation chosen by students according to the goal of the representation.

To illustrate, one student, Gemma, used an unannotated results tally to indicate her *understanding*. When she was asked to *explain* her findings, she provided the same tally but added: "*My answer 'What is the human proportion for the measurement you have' is 0.9 because it has come up the most times in the recording. It is pretty much the average*". Finally, when her task was to *convince* others, she changed her representation to an ordered dot plot (shown at Figure 2). Her annotation was changed to read: "*My answer to 'What is the human proportion for the Length of Foot to Length of Forearm?' is the range of normal is 0.9-1.2. My reason for this is, because 1.2 is very short of people (scores) but it still is 20% bigger than your foot but further than 1.2 I don't think it was really possible*".

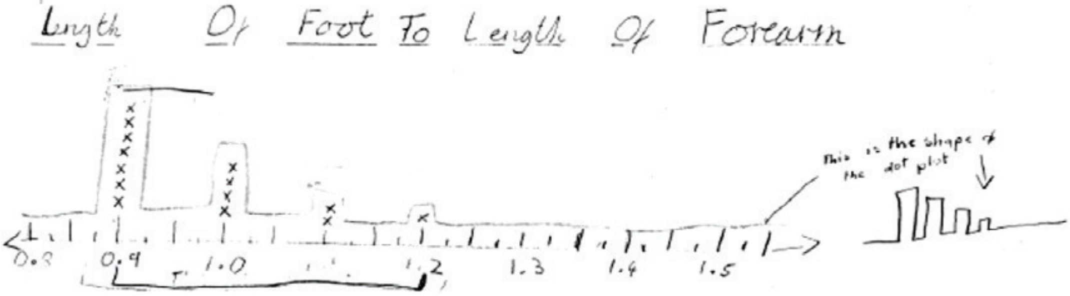

Figure 2. Gemma's Dot Plot Evidence.

From these final comments it is clear that Gemma is trying to anticipate questions or challenges to her data. By doing so, she is engaging with the data quite deeply and also applying proportional reasoning, envisaging what a ratio of 1:1.2 would mean in practical terms to enable her to defend her judgment.

Discussion and Conclusion

The purpose of this research was to make an exploratory study of inquiry-based argumentation practices through a focus on the role of evidence in students' communication. Berland and Reiser (2009) propose that students often engage in sensemaking (understanding) and articulation (explanation) of their developing knowledge, but that they do not consistently work to persuade others. The advantage of persuasion in the argumentation context is that it has the potential to lead to the development of increasingly robust knowledge, through enabling opportunities to critique, provide feedback, question, and challenge understandings.

The students in the research described here were provided with an open-ended inquiry problem with which they had to wrestle to determine initially a need for evidence, to envisage that evidence and then to put it forward in such a way as to convince others. The increasingly public nature of their explanations necessitated that they consider the applicability and strength of their findings; in other words the quality of their evidence (Sampson & Clarke, 2006), and be prepared to justify it.

The evidence provided by the students underwent significant change in practice. Initially, students were content with raw data, unorganised diagrams, or even a complete absence of evidence, in order to put forward their claim. However, as they began to see the need to explain to others, they became increasingly focussed on the effectiveness of their representations. Finally, when students were put in a position of being required to persuade others, signs of the students anticipating and attempting to circumvent audience critique became evident. Furthermore, students showed signs of engaging deeply with the mathematical content in order to address potential objections from their audience.

The research described here is limited to a heavily edited snapshot of students in a single classroom and there is no suggestion of generalising these findings. However, the results were particularly encouraging; not only were students able to provide claims and evidence, they had also begun to challenge others' ideas and to accept being challenged themselves. Perhaps more importantly, 'going public' enabled teacher insight into students' reasoning.

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